J.K. SHAH CLASSES

SOLUTION SET

MATHEMATICS & STATISTICS

SYJC PRELIUM – 02 – SET A

DURATION – 3 HR **MARKS – 80**

SECTION - I

Q1. (A)Attempt any six of the following **(12)**

01. Find X and Y if $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$; $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

SOLUTION

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \qquad X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \qquad X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \qquad 2Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \qquad Y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} \qquad Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

02. find $\frac{dy}{dx}$ if $y = \sin^{-1}\sqrt{1 - x^2}$

SOLUTION

Put $x = \cos\theta$

$$y = \sin^{-1}\sqrt{1 - \cos^2\theta}$$

$$y = \sin^{-1}\sqrt{\sin^2\theta}$$

$$y = \sin^{-1}(\sin\theta)$$

$$y = \theta$$

$$y = \cos^{-1} x$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1 - x^2}}$$

03. Find the value of k if the function

$$f(x) = \frac{\tan 7x}{2x} \quad ; \quad x \neq 0$$

$$= k \quad ; \quad x = 0 \qquad \text{is continuous at } x = 0$$

SOLUTION

Step 1

$\lim_{x \to 0} f(x)$

$$= \lim_{x \to 0} \frac{\tan 7x}{2x}$$

$$= \lim_{x \to 0} \frac{7}{2} \frac{\tan 7x}{7x}$$

$$= \frac{7}{2}(1)$$

$$= \frac{7}{2}$$

Step 2 :

f(0) = k ……… given

Step 3 :

Since f is continuous at x = 0

$$f(0) = \lim_{x \to 0} f(x)$$

$$k = 7/2$$

04. Write negations of the following statements

1. $\forall\, y \in N$, $y^2 + 3 \leq 7$

   Negation : $\exists\, y \in N$, such that $y^2 + 3 > 7$

2. if the lines are parallel then their slopes are equal

   Using : $\sim(P \rightarrow Q) \equiv P \wedge \sim Q$

   Negation : lines are parallel and their slopes are not equal

05. find elasticity of demand if the marginal revenue is Rs 50 and the price is Rs 75

SOLUTION

$$R_m = R_A\left(1 - \frac{1}{\eta}\right)$$

$$50 = 75\left(1 - \frac{1}{\eta}\right)$$

$$\frac{50}{75} = 1 - \frac{1}{\eta}$$

$$\frac{2}{3} = 1 - \frac{1}{\eta}$$

$$\frac{1}{\eta} = 1 - \frac{2}{3}$$

$$\frac{1}{\eta} = \frac{1}{3} \qquad \eta = 3$$

06. State which of the following sentences are statements . In case of statement , write down the truth value

a) Every quadratic equation has only real roots

ans : the given sentence is a logical statement . Truth value : F

b) $\sqrt{-4}$ is a rational number

ans : the given sentence is a logical statement . Truth value : F

07. Evaluate : $\int \frac{\sec^2 x}{\tan^2 x + 4}\, dx$

SOLUTION PUT $\tan x = t$

$\sec^2 x \, . \, dx = dt$

THE SUM IS

$$= \int \frac{1}{t^2 + 4}\, dt$$

$$= \int \frac{1}{t^2 + 2^2}\, dt$$

$$= \frac{1}{a} \tan^{-1} \frac{t}{a} + c$$

$$= \frac{1}{2} \tan^{-1} \frac{t}{2} + c$$

Resubs.

$$= \frac{1}{2} \tan^{-1}\left(\frac{\tan x}{2}\right) + c$$

08. if $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then find $|AB|$

SOLUTION

AB

$$= \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & 2+4 \\ 2+6 & 4+8 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ 8 & 12 \end{bmatrix}$$

$$|AB| = 4(12) - 8(6) = 48 - 48 = 0$$

Q2. (A)Attempt any TWO of the following (06)

01. $f(x) = \dfrac{3 - \sqrt{2x+7}}{x-1}$; $x \neq 1$

$= -1/3$; $x = 1$ Discuss continuity at x = 1

SOLUTION

STEP 1

$\lim\limits_{x \to 1} f(x)$

$$= \lim_{x \to 1} \frac{3 - \sqrt{2x+7}}{x-1}$$

$$= \lim_{x \to 1} \frac{3 - \sqrt{2x+7}}{x-1} \cdot \frac{3 + \sqrt{2x+7}}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{9 - (2x+7)}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{9 - 2x - 7}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{2 - 2x}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{2(1 - x)}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}}$$

$$= \lim_{x \to 1} \frac{-2(x - 1)}{x-1} \cdot \frac{1}{3 + \sqrt{2x+7}} \qquad x - 1 \neq 0$$

$$= \frac{-2}{3 + \sqrt{2 + 7}}$$

$$= \frac{-2}{3+3}$$

$$= \frac{-1}{3}$$

STEP 2 :

$f(1) = -1/3$ ……… given

STEP 3 :

$f(1) = \lim_{x \to 1} f(x)$; f is continuous at x = 1

02. Write the converse , inverse and the contrapositive of the statement

"The crops will be destroyed if there is a flood "

SOLUTION :

LET $P \to Q \equiv$ if there is a flood then the crops will be destroyed

CONVERSE : $Q \to P$

If the crops will be destroyed then there will be a flood

CONTRAPOSITIVE : $\sim Q \to \sim P$

If the crops will not be destroyed then there will be no flood

INVERSE : $\sim P \to \sim Q$

If there is no flood then the crops will not be destroyed

03. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{6x}{1 - 5x^2}\right)$

SOLUTION

$$y = \tan^{-1}\left(\frac{5x + x}{1 - 5x.x}\right)$$

$$y = \tan^{-1} 5x + \tan^{-1} x$$

$$\frac{dy}{dx} = \frac{1}{1 + 25x^2} . \frac{d}{dx}(5x) + \frac{1}{1 + x^2}$$

$$\frac{dy}{dx} = \frac{5}{1 + 25x^2} + \frac{1}{1 + x^2}$$

(B) Attempt any TWO of the following **(08)**

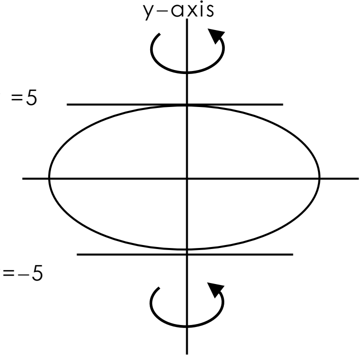
01. Find the volume of a solid obtained by the complete revolution of the ellipse

$$\frac{x^2}{36} + \frac{y^2}{25} = 1 \qquad \text{about Y – axis}$$

SOLUTION

STEP 1 :

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a^2 = 36 \ ; a = 6$$

$$b^2 = 25 \ , \ b = 5$$

STEP 2 :

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

$$\frac{x^2}{36} = 1 - \frac{y^2}{25}$$

$$\frac{x^2}{36} = \frac{25 - y^2}{25}$$

$$x^2 = \frac{36}{25}(25 - y^2)$$

STEP 3 :

$$V = \pi \int_{-5}^{5} x^2.dy$$

About y – axis

$$= \pi \int_{-5}^{5} \frac{36}{25}(25 - y^2).dy$$

$$= \frac{36\pi}{25} \int_{-5}^{5} (25 - y^2).dy$$

$$= \frac{36\pi}{25}\left[25y - \frac{y^3}{3}\right]_{-5}^{5}$$

$$= \frac{36\pi}{25}\left\{\left(125 - \frac{125}{3}\right) - \left(-125 + \frac{125}{3}\right)\right\}$$

$$= \frac{36\pi}{25}\left\{\left(\frac{375 - 125}{3}\right) - \left(\frac{-375 + 125}{3}\right)\right\}$$

$$= \frac{36\pi}{25}\left\{\left(\frac{250}{3}\right) - \left(\frac{-250}{3}\right)\right\}$$

$$= \frac{36\pi}{25}\left(\frac{500}{3}\right)$$

$= 240\,\pi$ cubic units

02. Evaluate : $\int \log(1 + x^2)\, dx$

$$= \int \log(1 + x^2).\, 1dx$$

$$= \log(1 + x^2)\int 1\, dx - \int\left(\frac{d}{dx}\log(1 + x^2)\int 1\, dx\right) dx$$

$$= \log(1 + x^2)\,.\, x - \int \frac{2x}{1 + x^2}.\, x \quad dx$$

$$= x.\log(1 + x^2) - 2\int \frac{x^2}{1 + x^2} \quad dx$$

$$= x.\log(1 + x^2) - 2\int \frac{1 + x^2 - 1}{1 + x^2}.\, dx$$

$$= x.\log(1 + x^2) - 2\int 1 - \frac{1}{1 + x^2} \quad dx$$

$$= x.\log(1 + x^2) - 2\left(x - \tan^{-1}x\right) + c$$

$$= x.\log(1 + x^2) - 2x + 2\tan^{-1}x + c$$

03. If Mr. Rao orders x cupboards , with demand function as

$$p = 2x + \frac{32}{x^2} - \frac{5}{x}$$

How many cupboards should he order for the most economical deal

SOLUTION

STEP 1 : COST

$$C = p.x$$

$$= \left(2x + \frac{32}{x^2} - \frac{5}{x}\right) . x$$

$$= 2x^2 + \frac{32}{x} - 5$$

STEP 2 :

$$\frac{dC}{dx} = 4x - \frac{32}{x^2} = 4x - 32x^{-2}$$

$$\frac{d^2C}{dx^2} = 4 + 64x^{-3}$$

$$= 4 + \frac{64}{x^3}.$$

STEP 3 :

$$\frac{dC}{dx} = 0$$

$$4x - \frac{32}{x^2} = 0$$

$$4x = \frac{32}{x^2}$$

$$4x^3 = 32$$

$$x^3 = 8 \qquad \therefore \ x = 2$$

STEP 4 :

$$\left.\frac{d^2C}{dx^2}\right|_{x = 2} = 4 + \frac{64}{2^3} > 0$$

Cost is minimum at x = 2

Mr. Rao must order 2 cupboards

Q3. **(A)Attempt any TWO of the following** **(06)**

01. Using ALGEBRA OF STATEMENTS , prove

$$p \wedge \left[(\sim p \vee q) \vee \sim q \right] \equiv p$$

Solution $p \wedge \left[(\sim p \vee q) \vee \sim q \right]$

$\equiv p \wedge \left[\sim p \vee (q \vee \sim q \right]$ ……… Associative Law

$\equiv p \wedge (\sim p \vee t)$ ……… Complement Law

$\equiv p \wedge t$ ……… Identity Law

$\equiv p$ ……… Identity Law

02. $f(x) = \dfrac{(e^{3x} - 1)^2}{x.\log(1 + 3x)}$; $x \neq 0$

$= 10$; $x = 0$ Discuss the continuity at $x = 0$

Solution :

Step 1

$$\lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{(e^{3x} - 1)^2}{x.\log(1 + 3x)}$$

Dividing Numerator & Denominator by x^2

$x \to 0$, $x \neq 0$, $x^2 \neq 0$

$$= \lim_{x \to 0} \frac{\frac{(e^{3x} - 1)^2}{x^2}}{\frac{x.\log(1 + 3x)}{x^2}}$$

$$= \lim_{x \to 0} \frac{\left(\frac{e^{3x} - 1}{x}\right)^2}{\frac{\log(1 + 3x)}{x}}$$

$$= \lim_{x \to 0} \frac{\left(3\,\frac{e^{3x} - 1}{3x}\right)^2}{\log(1 + 3x)^{\frac{1}{x}}}$$

$$= \lim_{x \to 0} \frac{\left(3 \quad \frac{e^{3x} - 1}{3x}\right)^2}{\log\left((1 + 3x)^{\frac{1}{3x}}\right)^3}$$

$$= \frac{(3.\log e)^2}{\log e^3}$$

$$= \frac{9}{3.\log e} \quad = \quad 3$$

Step 2 :

$f(0) = 10$ ……… given

Step 3 :

$$f(0) \neq \lim_{x \to 0} f(x)$$

$\therefore f$ is discontinuous at $x = 0$

Step 4 :

Removable Discontinuity

f can be made continuous at $x = 0$ by redefining it as

$$f(x) = \frac{(e^{3x} - 1)^2}{x.\log(1 + 3x)} \quad ; \quad x \neq 0$$

$$= 3 \quad ; \quad x = 0$$

03. if $\sin y = x.\sin(5 + y)$; prove that $\frac{dy}{dx} = \frac{\sin^2(5 + y)}{\sin a}$

SOLUTION

$$\sin y = x.\sin(5 + y)$$

$$x = \frac{\sin y}{\sin (5 + y)}$$

Differentiating wrt y

$$\frac{dx}{dy} = \frac{\sin (5 + y) \frac{d}{dy}\sin y - \sin y \frac{d}{dy}\sin (5 + y)}{\sin^2(5 + y)}$$

$$\frac{dx}{dy} = \frac{\sin (5 + y) . \cos y - \sin y . \cos (5 + y) \frac{d}{dy}(5 + y)}{\sin^2(5 + y)}$$

$$\frac{dx}{dy} = \frac{\sin (5 + y).\cos y - \cos (5 + y) . \sin y}{\sin^2(5 + y)}$$

$$\frac{dx}{dy} = \frac{\sin (5 + y - y)}{\sin^2(5 + y)}$$

$$\frac{dx}{dy} = \frac{\sin 5}{\sin^2(5 + y)}$$

Now $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

$\therefore$ $\frac{dy}{dx} = \frac{\sin^2(5+ y)}{\sin 5}$

(B) Attempt any TWO of the following **(08)**

01. $$\int_4^7 \frac{(11-x)^2}{x^2 + (11-x)^2}\,dx \qquad \ldots\ldots \quad \textbf{(1)}$$

$$\textbf{USING} \int_a^b f(x)dx = \int_b^b f(a+b-x)\,dx$$

$$I = \int_4^7 \frac{\left[11-(4+7-x)\right]^2}{(4+7-x)^2 + \left[11-(4+7-x)\right]^2}\,dx$$

$$I = \int_4^7 \frac{\left[11-(11-x)\right]^2}{(11-x)^2 + \left[11-(11-x)\right]^2}\,dx$$

$$I = \int_4^7 \frac{(11-11+x)^2}{(11-x)^2 + (11-11+x)^2}\,dx$$

$$I = \int_4^7 \frac{x^2}{(11-x)^2 + x^2}\,dx \qquad \ldots\ldots \quad \textbf{(2)}$$

(1) + (2)

$$2I = \int_4^7 \frac{(11-x)^2 + x^2}{(11-x)^2 + x^2}\,dx$$

$$2I = \int_4^7 1\,dx$$

$$2I = \left[x\right]_4^7$$

$$2I = 7 - 4$$

$$2I = 3$$

$$I = 3/2$$

02. $$\int \frac{x^2}{x^4 + 5x^2 + 6}\, dx$$

$$\int \frac{x^2}{(x^2 + 2)(x^2 + 3)}\, dx$$

SOLUTION

$$\frac{x^2}{(x^2 + 2)(x^2 + 3)} = \frac{A}{x^2 + 2} + \frac{B}{x^2 + 3}$$

x^2 = t (say)

$$\frac{t}{(t + 2)(t + 3)} = \frac{A}{t + 2} + \frac{B}{t + 3}$$

$$t = A(t + 3) + B(t + 2)$$

Put t = – 3

$$-3 = B(-3 + 2)$$

$$-3 = B(-1) \qquad \therefore\ B = 3$$

Put t = –2

$$-2 = A(-2 + 3)$$

$$-2 = A(1) \qquad \therefore\ A = -2$$

THEREFORE

$$\frac{t}{(t + 2)(t + 3)} = \frac{-2}{t + 2} + \frac{3}{t + 3}$$

HENCE

$$\frac{x^2}{(x^2 + 2)(x^2 + 3)} = \frac{-2}{x^2 + 2} + \frac{3}{x^2 + 3}$$

BACK IN THE SUM

$$= \int \frac{-2}{x^2 + 2} + \frac{3}{x^2 + 3}\, dx$$

$$= \int \frac{-2}{x^2 + \sqrt{2}^2} + \frac{3}{x^2 + \sqrt{3}^2}\, dx$$

$$= -2 \cdot \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + 3\ \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$

$$= -\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \sqrt{3} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$

03. $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix}$ Verify : $A.(\text{adj } A) = (\text{adj } A).A = |A|.I$

COFACTOR'S

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = 1(0-0) = 0$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -1(9+2) = -11$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 1(0-0) = 0$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -1(-3-0) = 3$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1(3-2) = 1$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -1(0+1) = -1$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 1(2-0) = 2$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -1(-2-6) = 8$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 1(0+3) = 3$$

COFACTOR MATRIX OF A

$$\begin{pmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{pmatrix}$$

ADJ A **=** TRANSPOSE OF THE COFACTOR MATRIX

$$= \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix}$$

|A|

$= 1(0 + 0) + 1(9 + 2) + 2(0 - 0)$

$= 11$

LHS 1

$=$ A.(adj A)

$$= \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 + 11 + 0 & 3 - 1 - 2 & 2 - 8 + 6 \\ 0 - 0 - 0 & 9 + 0 + 2 & 6 + 0 - 6 \\ 0 - 0 + 0 & 3 + 0 - 3 & 2 + 0 + 9 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

LHS 2

$=$ (adj A) . A

$$= \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 + 9 + 2 & 0 + 0 + 0 & 0 - 6 + 6 \\ -11 + 3 + 8 & 11 + 0 + 0 & -22 - 2 + 24 \\ 0 - 3 + 3 & 0 - 0 + 0 & 0 + 2 + 9 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

RHS

$=$ |A|.I

$$= 11 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

HENCE A.(adj A) = (adj A).A = |A|.I

SECTION – II

Q4. (A)Attempt any six of the following (12)

01. Find correlation coefficient between x and y for the following data

$n = 100$, $\overline{x} = 62$, $\overline{y} = 53$, $\sigma x = 10$, $\sigma y = 12$, $\Sigma(x - \overline{x})(y - \overline{y}) = 8000$

SOLUTION $r = \dfrac{cov(x,y)}{\sigma x \,.\, \sigma y}$

$$= \dfrac{\dfrac{\Sigma(x - \overline{x})(y - \overline{y})}{n}}{\sigma x \,.\, \sigma y}$$

$$= \dfrac{\dfrac{8000}{100}}{10.12}$$

$$= \dfrac{80}{10.12}$$

$$= \dfrac{2}{3}$$

02. a building is insured for 80% of its value . The annual premium at 70 paise percent amounts to ₹ 2,800 . Fire damaged the building to the extent of 60% of its value . How much amount for damage can be claimed under the policy

SOLUTION

Property value = ₹ x

Insured value $= \dfrac{80x}{100} = \dfrac{4x}{5}$

Rate of premium = 70 paise percent

= 0.70%

Premium = ₹ 2800

$2800 = \dfrac{0.70}{100} \times \dfrac{4x}{5}$

$2800 = \dfrac{7}{1000} \times \dfrac{4x}{5}$

$2800 = \dfrac{28x}{5000}$

$x = 100 \times 5000$

$x = 5,00,000$

Property value = ₹ 5,00,000

Loss $= \frac{60}{100} \times 5,00,000$

$= ₹\ 3,00,000$

Claim = 80% of loss

$= \frac{80}{100} \times 3,00,000$

$= ₹\ 2,40,000$

03. The coefficient of rank correlation for a certain group of data is 0.5 . If $\Sigma d^2 = 42$, assuming no ranks are repeated ; find the no. of pairs of observation

SOLUTION

$R = 0.5 \quad ; \quad \Sigma d^2 = 42$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$0.5 = 1 - \frac{6(42)}{n(n^2 - 1)}$$

$$\frac{6(42)}{n(n^2 - 1)} = 1 - 0.5$$

$$\frac{6(42)}{n(n^2 - 1)} = 0.5$$

$$\frac{6(42)}{n(n^2 - 1)} = \frac{1}{2}$$

$$n(n^2 - 1) = 6 \times 42 \times 2$$

$$n(n^2 - 1) = 3 \times 2 \times 3 \times 2 \times 7 \times 2$$

$$(n - 1).n.(n + 1) = 7 \times 8 \times 9$$

On comparing , n = 8

04. Maya and Jaya started a business by investing equal amount . After 8 months Jaya withdrew her amount and Priya entered the business with same amount of capital . At the end of the year there was a profit of ₹ 13,200 . Find their share of profit

SOLUTION

STEP 1 :

Profits will be shared in the

'RATIO OF PERIOD OF INVESTMENT'

	MAYA		JAYA		PRIYA
=	12	:	8	:	4

TOTAL = 24

STEP 2 :

PROFIT = ₹ 13,200

Maya's share of profit $= \frac{12}{\not{24}_{2}} \times \not{13,200}^{1100} =$ ₹ 6,600

Jaya's share of profit $= \frac{8}{\not{24}_{2}} \times \not{13,200}^{1100} =$ ₹ 4,400

Priya'sshare of profit $= \frac{4}{\not{24}_{2}} \times \not{13,200}^{1100} =$ ₹ 2,200

05. Calculate CDR for district A and B and compare

Age Group (Years)	DISTRICT A		DISTRICT B	
	NO. OF PERSONS **P**	NO. OF DEATHS **D**	NO. OF PERSONS **P**	NO. OF DEATHS **D**
0 – 10	1000	18	3000	70
10 – 55	3000	32	7000	50
Above 55	2000	41	1000	24
	ΣP = 6000	**ΣD = 91**	**ΣP = 11000**	**ΣD = 144**

CDR(A) $= \frac{\Sigma D}{\Sigma P} \times 1000$

$= \frac{91}{6000} \times 1000$

$= 15.17$

(DEATHS PER THOUSAND)

CDR(B) $= \frac{\Sigma D}{\Sigma P} \times 1000$

$= \frac{144}{11000} \times 1000$

$= 13.09$

(DEATHS PER THOUSAND)

COMMENT : CDR(B) < CDR(A) . HENCE DISTRICT B IS HEALTHIER THAN DISTRICT A

06. the probability of defective bolts in a workshop is 40% . Find the mean and variance of defective bolts out os 10 bolts

SOLUTION $n = 10$,

r,v,x = no of defective bolts

p = probability of defective bolt $= \frac{40}{100} = \frac{2}{5}$

$q = 1 - p = \frac{3}{5}$

$X \sim B(10, {}^{2}/_{5})$

Mean $= np = 10 \times \frac{2}{5} = 5$

Variance $= npq = 10 \times \frac{2}{5} \times \frac{3}{5} = 2.4$

07. The ratio of incomes of Salim & Javed was 20:11 . Three years later income of Salim has increased by 20% and income of Javed was increased by ₹ 500 . Now the ratio of their incomes become 3 : 2 . Find original incomes of Salim and Javed

SOLUTION

Let income of Salim = 20x

Income of Javed = 11x

As per the given condition

$$\frac{20x + \frac{20}{100}(20x)}{11x + 500} = \frac{3}{2}$$

$$\frac{20x + 4x}{11x + 500} = \frac{3}{2}$$

$$\frac{24x}{11x + 500} = \frac{3}{2}$$

$$48x = 33x + 1500$$

$$x = 100$$

$\therefore$

Salim's original income = 20(100) = ₹ 2000

Javed's original income = 11(100) = ₹ 1100

08. for an immediate annuity paid for 3 years with interest compounded at 10% p.a. its present value is ₹ 10,000 . What is the accumulated value after 3 years ($1.1^3 = 1.331$)

SOLUTION $A = P(1 + i)^n$

$= 10000(1 + 0.1)^3$

$= 10000(1.1)^3$

$= 10000(1.331)$

= ₹ 13,310

Q5. (A)Attempt any Two of the following **(06)**

01. a new treatment for baldness is known to be effective in 70% of the cases treated . Four bald members from different families are treated . Find the probability that

(i) at least one member is successfully treated (ii) exactly 2 members are successfully treated

SOLUTION

4 bald members from different families are treated , n = 4

For a trial Success – a defective pen

p – probability of success = 70/100 = 7/10

q – probability of failure = 1 – 7/10 = 3/10

r.v. X – no of successes = 0 , 1 , 2 , 3 , 4 **X ~ B (4 , 7/10)**

a) P(at least one member is successfully treated)

$= P(X \geq 1)$

$= P(1) + P(2) + \ldots\ldots + P(4)$

$= 1 - P(0)$

$= 1 - {}^4C_0 \,.\, p^0 \,.\, q^4$

$= 1 - {}^4C_0 \left(\frac{7}{10}\right)^0 \left(\frac{3}{10}\right)^4$

$= 1 - \frac{1 \,.\, 1 \,.\, 81}{10000}$

$= 1 - 0.0081$

$= 0.9919$

b) P(exactly 2 members are successfully treated)

$= P(X = 2)$

$= {}^4C_2 \,.\, p^2 \,.\, q^2$

$= {}^4C_2 \left(\frac{7}{10}\right)^2 \left(\frac{3}{10}\right)^2$

$= \frac{6 \,.\, 49 \,.\, 9}{10^4}$

$= \frac{2646}{10000} = 0.2646$

02. Compute rank correlation coefficient for the following data

Rx :	1	2	3	4	5	6
Ry :	6	3	2	1	4	5

SOLUTION

x	y	d = |x − y|	d^2
1	6	5	25
2	3	1	1
3	2	1	1
4	1	3	9
5	4	1	1
6	5	1	1
			$\Sigma d^2 = 38$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(38)}{6(36 - 1)}$$

$$= 1 - \frac{38}{35}$$

$$= \frac{-3}{35}$$

$$= -0.086$$

03. the income of the agent remains unchanged though the rate of commission is increased from 5% to 6.25% . Find the percentage reduction in the value of the business

SOLUTION

Let initial sales = ₹ 100

Rate of commission = 5%

∴ Commission = ₹ 5

Let the new sales = ₹ x

Rate of commission = 6.25%

∴ Commission = $\frac{6.25x}{100}$

Since the income of the broker remains unchanged

$$\frac{6.25\,x}{100} = 5$$

$$x = \frac{5 \times 100 \times 100}{625}$$

$$x = 80$$

∴ new sales = ₹ 80

Hence percentage reduction in the value of the business = 20%

(B) Attempt any Two of the following **(08)**

01. A warehouse valued at ₹ 10,000 contained goods worth ₹ 60,000 . The warehouse was insured against fire for ₹ 4,000 and the goods to the extent of 90% of their value . A fire broke out and goods worth ₹ 20,000 were completely destroyed , while the remainder was damaged and reduced to 80% of its value . The damage to the warehouse was to the extent of ₹ 2,000 . Find the total amount that can be claimed

SOLUTION :

WAREHOUSE

Property value = ₹ 10,000

Insured value = ₹ 4,000

Loss = ₹ 2,000

Claim $= \frac{\text{insured val.}}{\text{Property val.}} \times \text{loss}$

$= \frac{4,000}{10,000} \times 2,000$

= ₹ 800

STOCK IN WAREHOUSE

Value of stock = ₹ 60,000

Insured value = 90% of the stock

Loss

Note : Since the remainder was reduced to 80% of its value the loss on it is 20%

$= 20,000 + \frac{20}{100}(60,000 - 20,000)$

$= 20,000 + \frac{20}{100}(40,000)$

= 20, 000 + 8,000

= ₹ 28,000

Since 90% of the stock was insured

Claim = 90% of loss

$= \frac{90}{100} \times 28,000$

= ₹ 25,200

Hence

Total claim = 800 + 25,200

= ₹ 26,000

02. X : 6 2 10 4 8

Y : 9 11 ? 8 7

Arithmetic means of X and Y series are 6 and 8 respectively . Calculate correlation coefficient

SOLUTION :

$$\bar{y} = \frac{\Sigma y}{n} \qquad 8 = \frac{9 + 11 + b + 8 + 7}{5}$$

$$40 = 35 + b \qquad b = 5$$

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
6	9	0	1	0	1	0
2	11	–4	3	16	9	–12
10	5	4	–3	16	9	–12
4	8	–2	0	4	0	0
8	7	2	–1	4	1	–2
30 Σx	40 Σy	0 $\Sigma(x-\bar{x})$	0 $\Sigma(y-\bar{y})$	40 $\Sigma(x-\bar{x})^2$	20 $\Sigma(y-\bar{y})^2$	–26 $\Sigma(x-\bar{x})(y-\bar{y})$
$\bar{x} = 6$ $\bar{y} = 8$						

$$r = \frac{\Sigma (x-\bar{x}).(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\ \sqrt{\Sigma(y-\bar{y})^2}.}$$

$$r = \frac{-26}{\sqrt{40} \times \sqrt{20}}$$

$$r = \frac{-26}{\sqrt{40 \times 20}}$$

$$r' = \frac{26}{\sqrt{40 \times 20}}$$

taking log on both sides

$$\log r' = \log 26 - \frac{1}{2}\left(\log 40 + \log 20\right)$$

$$\log r' = 1.4150 - \frac{1}{2}[1.6021 + 1.3010]$$

$$\log r' = 1.4150 - \frac{1}{2}(2.9031)$$

$$\log r' = 1.4150 - 1.4516$$

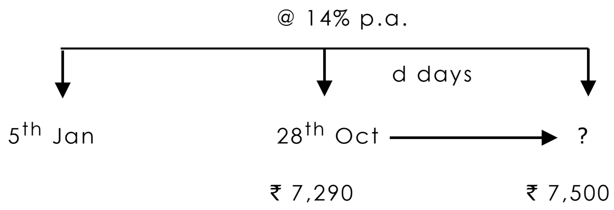
$$\log r' = \bar{1}.9634$$

$$r' = AL(\bar{1}.9634) = 0.9191$$

$$r = -0.9191$$

03. a bill of ₹ 7,500 was discounted for ₹ 7290 at a bank on 28th October 2006 . If the rate of interest was 14% p.a. , what is the legal due date

SOLUTION



STEP 1:

Let Unexpired period = d days

STEP 2 :

B.D. = F.V. – C.V.

= 7,500 – 7,290

= ₹ 210

STEP 3 :

B.D. = Interest on F.V. for 'd' days @ 14% p.a.

$$210 = \overset{15}{\cancel{7500}} \times \frac{d}{\underset{73}{\cancel{365}}} \times \frac{14}{\cancel{100}}$$

$$d = \frac{210 \times 73}{15 \times 14}$$

d = 73 days

STEP 4:

Legal Due date

= 28th Oct + 73 days

	OCT		NOV		DEC		JAN
=	3	+	30	+	31	+	9

= 9th January 2007

Q6. (A) Attempt any Two of the following (06)

01. The number of complaints which a bank manager receives per day is a Poisson random variable with parameter m = 4 . Find the probability that the manager will receive at most two complaints on any given day ($e^{-4} = 0.0183$)

SOLUTION

m = average number of complaints a bank manager receives per day = 4

r.v $X \sim P(4)$

P(at most two complaints on any given day)

$= P(x \leq 2)$

$= P(0) + P(1) + P(2)$

$= \frac{e^{-4}.4^0}{0!} + \frac{e^{-4}.4^1}{1!} + \frac{e^{-4}.4^2}{2!}$ Using $P(x) = \frac{e^{-m}.m^x}{x!}$

$= e^{-4}.\left(\frac{1}{1} + \frac{4}{1} + \frac{16}{2}\right)$

$= 0.0183\,(1 + 4 + 8)$

$= 0.0183(13)$

$= 0.2379$

02. Suppose X is a random variable with pdf

$$f(x) = \frac{c}{x} \quad ; \quad 1 < x < 3 \quad ; \quad c > 0$$

Find c & E(X)

i) $$\int_1^3 \frac{c}{x} \, dx = 1$$

$$c \int_1^3 \frac{1}{x} \, dx = 1$$

$$c \Big[\log x\Big]_1^3 = 1$$

$$c \Big[\log 3 - \log 1\Big] = 1$$

$$c \log 3 = 1$$

$$c = \frac{1}{\log 3}$$

Hence X is a r.v. with pdf

$$f(x) = \frac{1}{x.\log 3} \quad ; \quad 1 < x < 3$$

ii) $$E(x) = \int_1^3 x.f(x) \, dx$$

$$= \int_1^3 x. \frac{1}{x.\log 3} \, dx$$

$$= \int_1^3 \frac{1}{\log 3} \, dx$$

$$= \left[\frac{x}{\log 3}\right]_1^3$$

$$= \left(\frac{3}{\log 3}\right) - \left(\frac{1}{\log 3}\right) = \frac{2}{\log 3}$$

03. In a factory there are six jobs to be performed , each of which should go through machines A and B in the order A – B . Determine the sequence for performing the jobs that would minimize the total elapsed time T . Find T and the idle time on the two machines

Job	J_1	J_2	J_3	J_4	J_5	J_6
M_A	1	3	8	5	6	3
M_B	5	6	3	2	2	10

Step 1 : Finding the optimal sequence

Min time = 1 on job J_1 on machine M_1 . Place the job at the start of the sequence

J_1					

Next min time= 2 on jobs J_4 & J_5 on machine M_B . Place the jobs at the end of the sequence randomly

J_1				J_4	J_5

Placed Randomly

Next min time = 3 on jobs J_2 & J_6 on machine M_A and on job J_3 on machine M_B respectively . Place J_2 & J_6 at the start next to J_1 randomly and J_3 at the end next to J_4

J_1	J_2	J_6	J_3	J_4	J_5

Placed Randomly

OPTIMAL SEQUENCE

J_1	J_2	J_6	J_3	J_4	J_5

Step 2 : Work table

According to the optimal sequence

Job	**J_1**	**J_2**	**J_6**	**J_3**	**J_4**	**J_5**	**total process time**	
M_A	1	3	3	8	5	6	=	26 hrs
M_B	5	6	10	3	2	2	=	28 hrs

WORK TABLE

	MACHINES				Idle time
	M_A		M_B		
JOBS	IN	OUT	IN	OUT	on M_B
J_1	0	1	1	6	1
J_2	1	4	6	12	
J_6	4	7	12	22	
J_3	7	15	22	25	
J_4	15	20	25	27	
J_5	20	26	27	29	

Step 3 :

Total elapsed time T = 29 hrs

Idle time on M_A = T − (sum of processing time of all 6 jobs on M1)

= 29 − 26

= 3 hrs

Idle time on M_B = T − (sum of processing time of all 6 jobs on M2)

= 29 − 28

= 1 hr

Step 4 : All possible optimal sequences :

$J_1 - J_2 - J_6 - J_3 - J_4 - J_5$

OR

$J_1 - J_6 - J_2 - J_3 - J_4 - J_5$

OR

$J_1 - J_2 - J_6 - J_3 - J_5 - J_4$

OR

$J_1 - J_6 - J_2 - J_3 - J_5 - J_4$

(B) Attempt any Two of the following **(08)**

01. a pharmaceutical company has four branches , one at each city A , B , C and D . A branch manager is to be appointed one at each city , out of four candidates P , Q , R and S . The monthly business depends upon the city and effectiveness of the branch manager in that city

CITY

		A	B	C	D
	P	11	11	9	9
BRANCH	Q	13	16	11	10
MANAGER	R	12	17	13	8
	S	16	14	16	12

MONTHLY BUSINESS (IN LACS)

Which manager should be appointed at which city so as to get maximum total monthly business .

6	6	8	8
4	1	6	7
5	0	4	9
1	3	1	5

Subtracting all the elements in the matrix from its maximum
The matrix can now be solved for 'MINIMUM'

0	0	2	2
3	0	5	6
5	0	4	9
0	2	0	4

Reducing the matrix using 'ROW MINIMUM'

0	0	2	0
3	0	5	4
5	0	4	7
0	2	0	2

Reducing the matrix using 'COLUMN MINIMUM'

[0]	~~0~~	2	~~0~~
3	[0]	5	4
5	~~0~~	4	7
~~0~~	2	[0]	2

– Allocation using 'SINGLE ZERO ROW-COLUMN METHOD'
– Allocation incomplete (3^{rd} row unassigned)

	[0]	~~0~~	2	~~0~~
√	3	[0]	5	4
√	5	~~0~~	4	7
	~~0~~	2	[0]	2
		√		

– Drawing min. no. of lines to cover all '0's

0	3	2	0	Revise the matrix
0	0	2	1	Reducing all the uncovered elements by its minimum and adding the same at the intersection
2	0	1	4	
0	4	0	2	

~~0~~	3	2	[0]	– Reallocation using 'SINGLE ZERO ROW-COLUMN METHOD'
[0]	~~0~~	2	1	– Since all rows contain an 'assigned zero' , the assignment problem is complete
2	[0]	1	4	
~~0~~	4	[0]	2	

Optimal Assignment : P – D ; Q – A; R – B ; S – C

Maximum business = 9 + 13 + 17 + 16 = 55(lacs)

02. Information on vehicles (in thousands) passing through seven different highways during a day (X) and number of accidents reported (Y) is given as

$\Sigma x = 105$; $\Sigma y = 409$; $\Sigma x^2 = 1681$; $\Sigma y^2 = 39350$; $\Sigma xy = 8075$.

Obtain linear regression of Y on X

SOLUTION

$$\bar{x} = \frac{\Sigma x}{n} = \frac{105}{7} = 15$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{409}{7} = 58.43$$

$$byx = \frac{n\Sigma xy - \Sigma x.\Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{7(8075) - (105)(409)}{7(1681) - (105)^2}$$

$$= \frac{56525 - 42945}{11767 - 11025}$$

$$= \frac{13580}{742}$$

$$= 18.30$$

LOG CALC

4.1329
– 2.8704
AL 1.2625
18.30

Equation

$$y - \bar{y} = byx\,(x - \bar{x})$$

$$y - 58.43 = 18.30(x - 15)$$

$$y - 58.43 = 18.30x - 274.5$$

$$y = 18.30x - 274.50 + 58.43$$

$$y = 18.30x - 216.07$$

03. Minimize $z = 3x_1 + 2x_2$

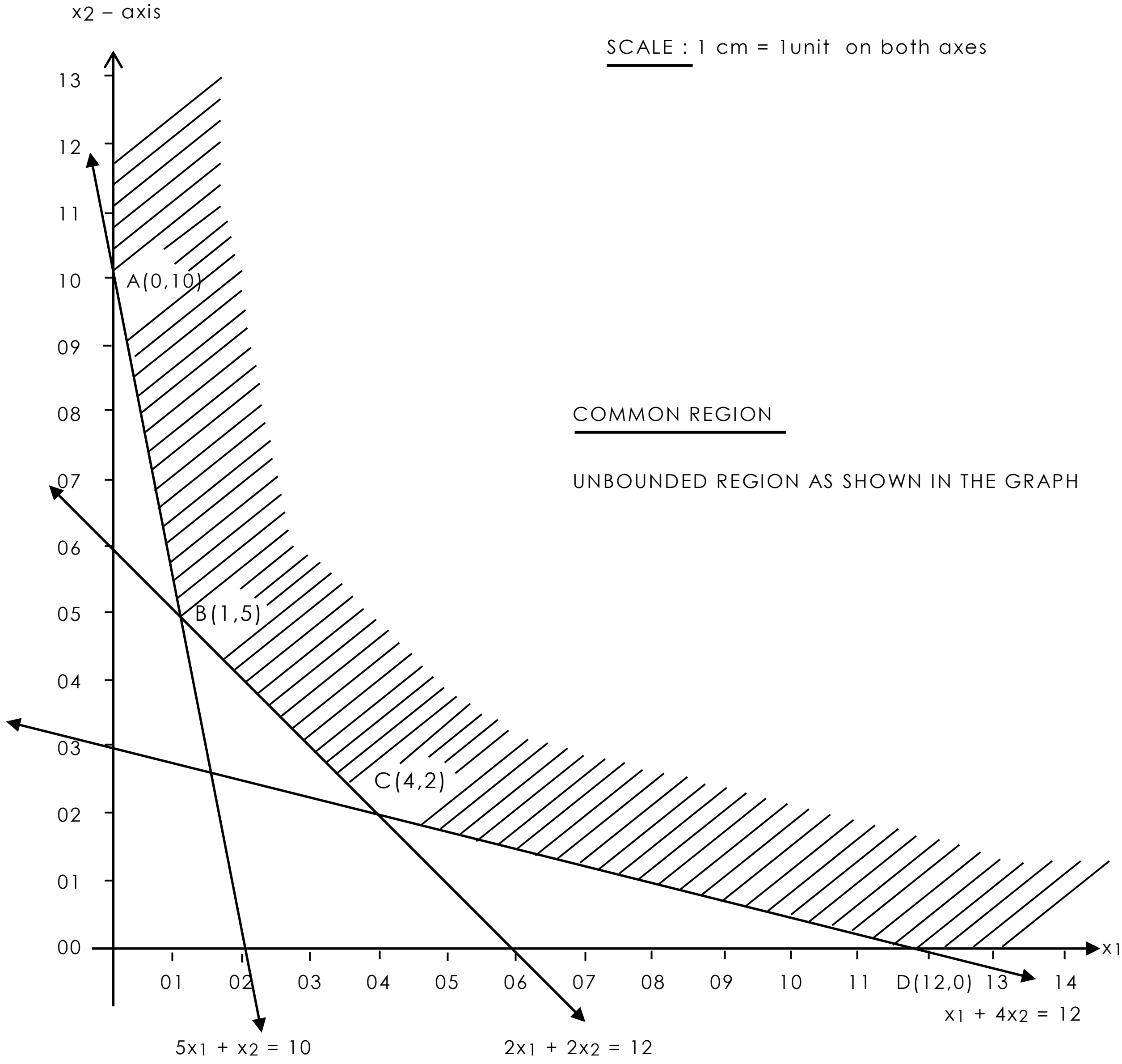
subject to : $5x_1 + x_2 \geq 10$; $2x_1 + 2x_2 \geq 12$; $x_1 + 4x_2 \geq 12$; $x_1, x_2 \geq 0$

STEP 1 :

$5x_1 + x_2 \geq 10$	$5x_1 + x_2 = 10$	Put (0,0) in $5x_1 + x_2 \geq 10$
	cuts x_1 – axis at (2,0)	$0 \geq 10$
	cuts x_2 – axis at (0,10)	SS : non – origin side
$2x_1 + 2x_2 \geq 12$	$2x_1 + 2x_2 = 12$	Put (0,0) in $2x_1 + 2x_2 \geq 12$
	cuts x_1 – axis at (6,0)	$0 \geq 12$
	cuts x_2 – axis at (0,6)	SS : non – origin side
$x_1 + 4x_2 \geq 12$	$x_1 + 4x_2 = 12$	Put (0,0) in $x_1 + 4x_2 \geq 12$
	cuts x_1 – axis at (12,0)	$0 \geq 12$
	cuts x_2 – axis at (0,3)	SS : non – origin side
$x_1, x_2 \geq 0$		SS : I Quadrant

STEP 2 :

SCALE : 1 cm = 1unit on both axes



COMMON REGION

UNBOUNDED REGION AS SHOWN IN THE GRAPH

STEP 3 :

CORNERS	$z = 3x_1 + 2x_2$		
A(0,10)	3(0) + 2(10) =	0 + 20	= 20
B(1, 5)	3(1) + 2(5) =	3 + 10	= 13
C(4,2)	3(4) + 2(2) =	12 + 4	= 16
D(12,0)	3(12) + 2(0) =	36 + 0	= 36

OPTIMAL SOLUTION : Zmin = 13 at (1,5)